Making 3D Binary Digital Images Well-Composed

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Introduction

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- a **probabilistic bound** for the (theoretical) effectiveness of our algorithm, and
- several **examples** of the application of our algorithm to magnetic resonance (MR) images.

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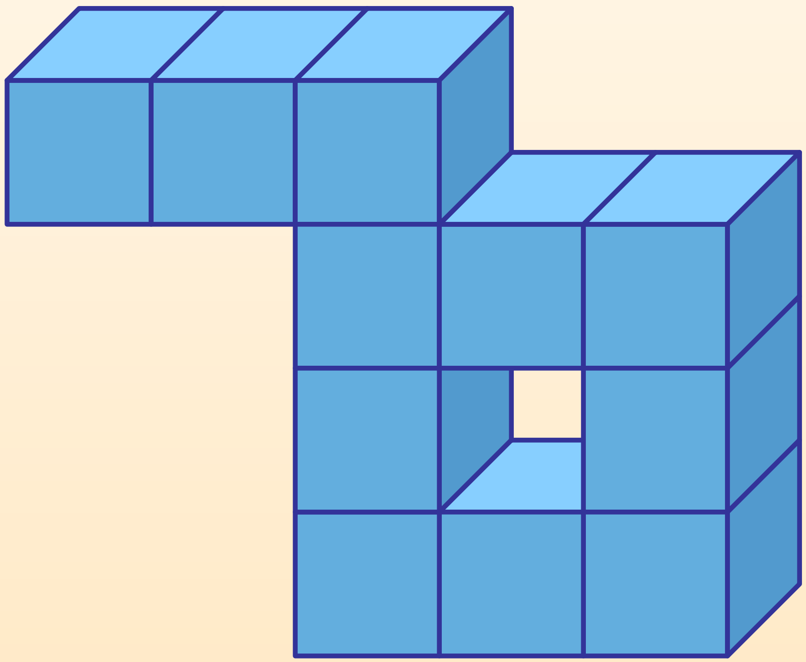

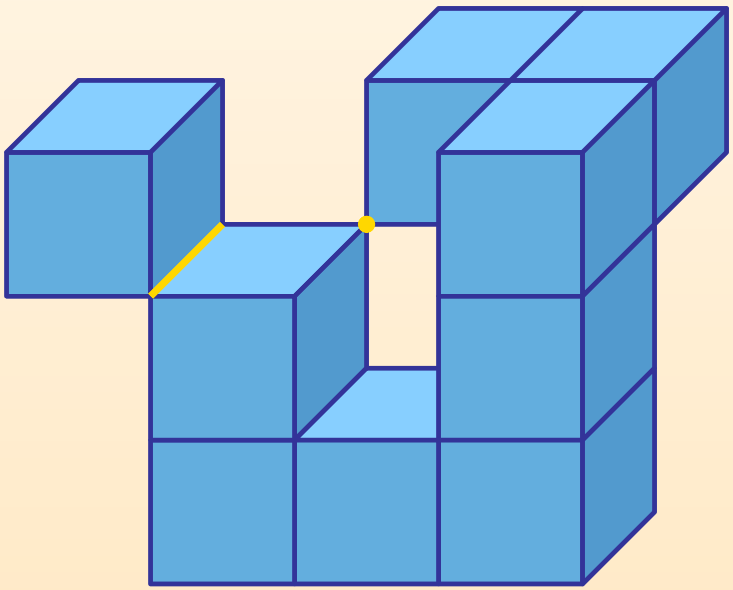

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What if a 3D binary digital image is not well-composed?

- A previous result shows that the **digitization process** that gave rise to the (ill-composed) image is not **topology-preserving.** Otherwise, the image would be well-composed!

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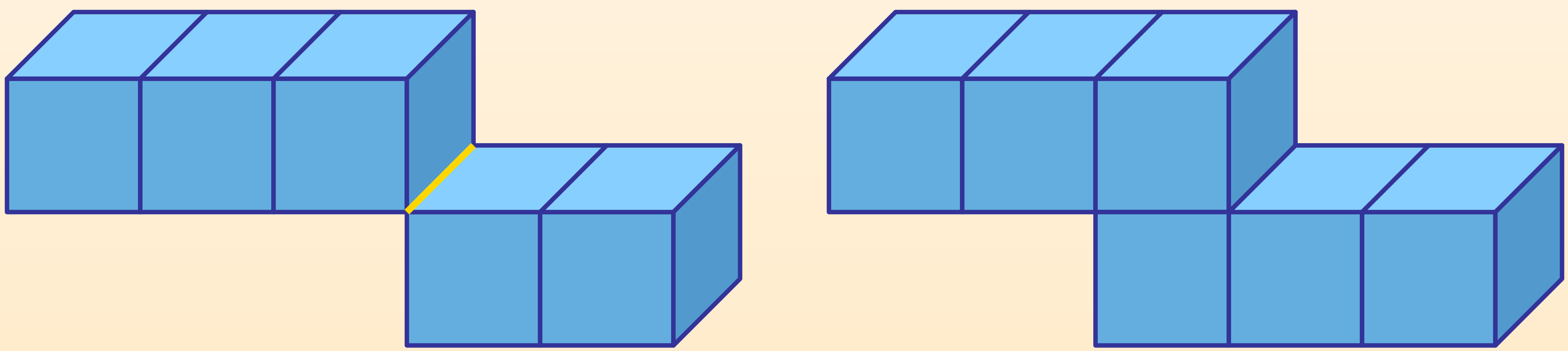

Before After

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What do we mean by ‘‘similar’’? How do we measure similarity?

We now describe our algorithm for repairing ill-composed images.

The Repairing Algorithm

Our repairing algorithm makes use of the fact (proved elsewhere) that a 3D digital binary image has the well-composedness property if and only if the image does not contain any instance of the following two **critical configurations** (up to reflections and rotations):

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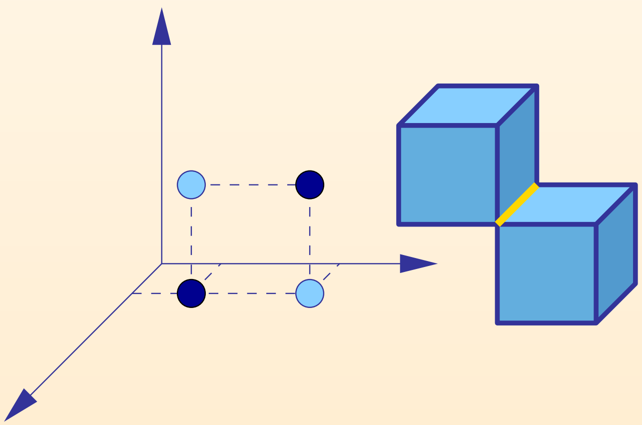
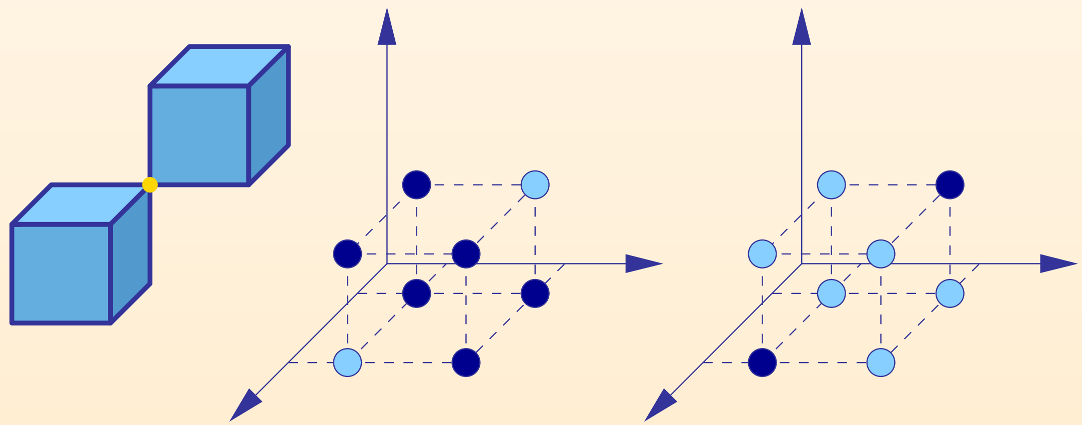


Critical Configuration 1 (C1)



Critical Configuration 2 (C2)

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There are two main issues related to this strategy:

- Can our repairing algorithm always generate a well-composed image?
- If so, which background points should be converted into foreground points so that all critical configurations are removed? And how similar are the input and the output images?

The Repairing Algorithm

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We can easily find such a smallest set by enumerating all possible subsets of background points in decreasing order of cardinality.

The problem with the above approach is that its performance can be really poor, as there are 2^n subsets of background points in an image with n background points.

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The key idea behind our approach is to exploit the ''locality'' of critical configurations.

Let (D, X) denote a 3D binary digital image, where D is the set of points of the image (i.e, the image domain), and $X \subseteq D$ is the set of foreground points of the image.

Let $P \subseteq (D \setminus X)$ denote the set of background points obtained by our algorithm, where $D \setminus X$ is the complement set of X with respect to D, i.e., the set of background points of (D, X).

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Let $(D, X \cup P^i)$ denote the 3D binary digital image obtained from (D, X) by converting the set P^i of background points of (D, X) into foreground points.

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At each iteration i of the algorithm, one or more points from $D \setminus P^i$ is inserted into P^i.

The insertion of a point from $D \setminus P^i$ into P^i eliminates at least one instance of a critical configuration of $(D, X \cup P^i)$.

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- Let $i = 1$ and let $P = \emptyset$.
- While $(D, X \cup P^i)$ is not well-composed do
  - ▷ *Find all instances of (C1) and (C2) in* $(D, X \cup P^i)$ *and eliminate them.*
  - ▷ *Increment* i *by* 1
- Output $(D, X \cup P)$.

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- Let $i = 1$ and let $P = \emptyset$.
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  - ▷ *Find all instances of (C1) and (C2) in* $(D, X \cup P^i)$ *and eliminate them.*
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- Output $(D, X \cup P)$.

Note that, for $i \geq 2$, if $(D, X \cup P^i)$ is not well-composed, then all instances of (C1) and (C2) in $(D, X \cup P^i)$ were created during the elimination of all instances of (C1) and (C2) in $(D, X \cup P^{i-1})$.

The Repairing Algorithm

To find all instances of (C1) and (C2) in $(D, X \cup P^1) = (D, X \cup P)$, we scan the entire set D and examine every 2×2 and $2 \times 2 \times 2$ subsets of points of D.

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For $i \geq 2$, we do not scan D again. Instead, we keep track of all new critical configurations created during the elimination of all instances of (C1) and (C2) in $(D, X \cup P^{i-1})$, as they are the only critical configurations of $(D, X \cup P^i)$.

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Each critical configuration of $(D, X \cup P^i)$ is eliminated by the insertion of **exactly one** background point from $D \setminus P^i$ into P^i.

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The choice of a background point from $D \setminus P^i$ to insert into P^i is the key step of our algorithm.

The Repairing Algorithm

How do we choose a point from $D \setminus P^i$ to insert into P^i?

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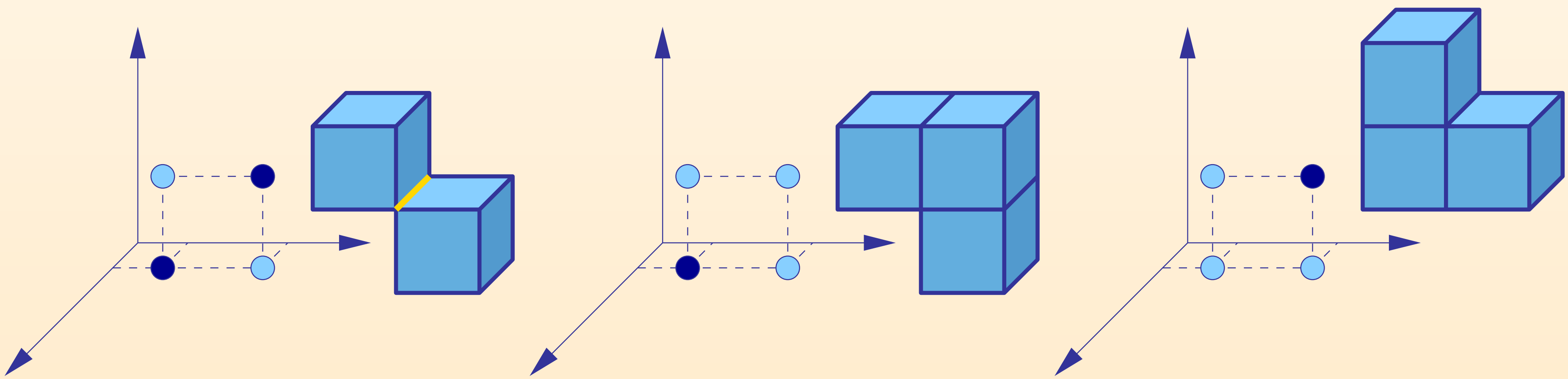

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Unlike instances of (C1), an instance of (C2) can be of one of two types:

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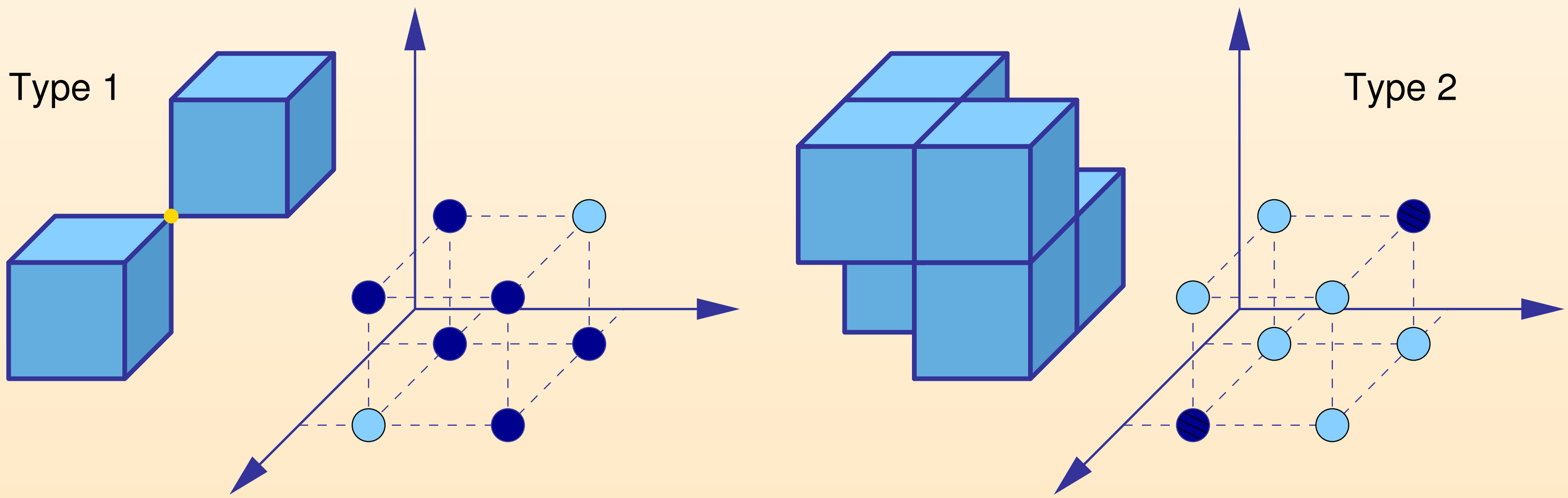
Unlike instances of (C1), an instance of (C2) can be of one of two types:

- Type 1: There are **six background** points and **two foreground** points in the instance of (C2).
- Type 2: There are **two background** points and **six foreground** points in the instance of (C2).



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By observing the picture below, we see that a critical configuration 2 of type 1 can **only be removed if** one (or more) of its six background points is made into a foreground point.

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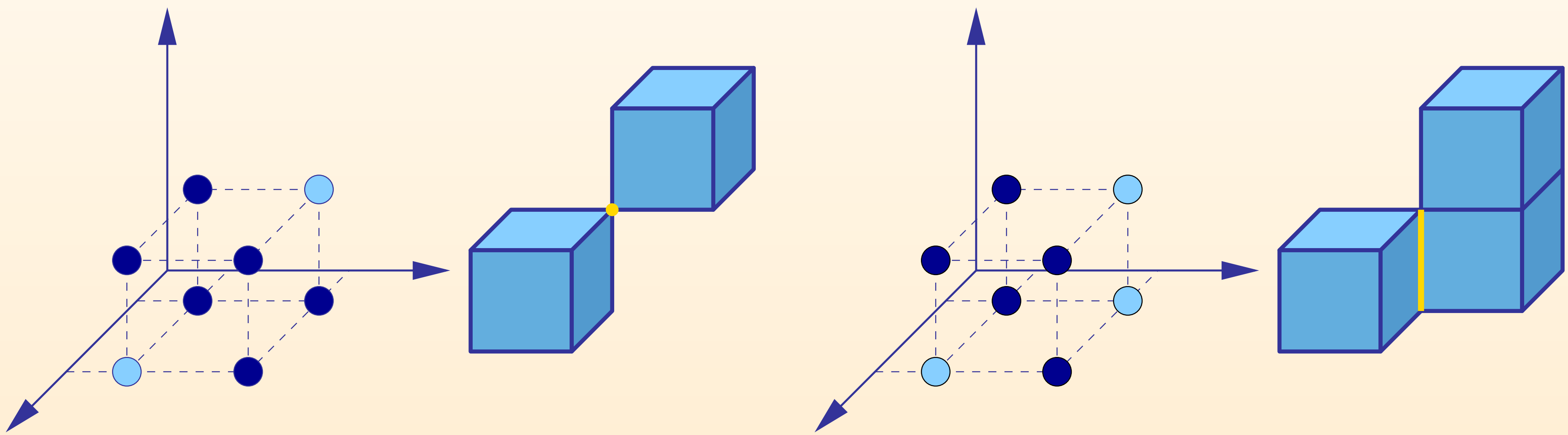

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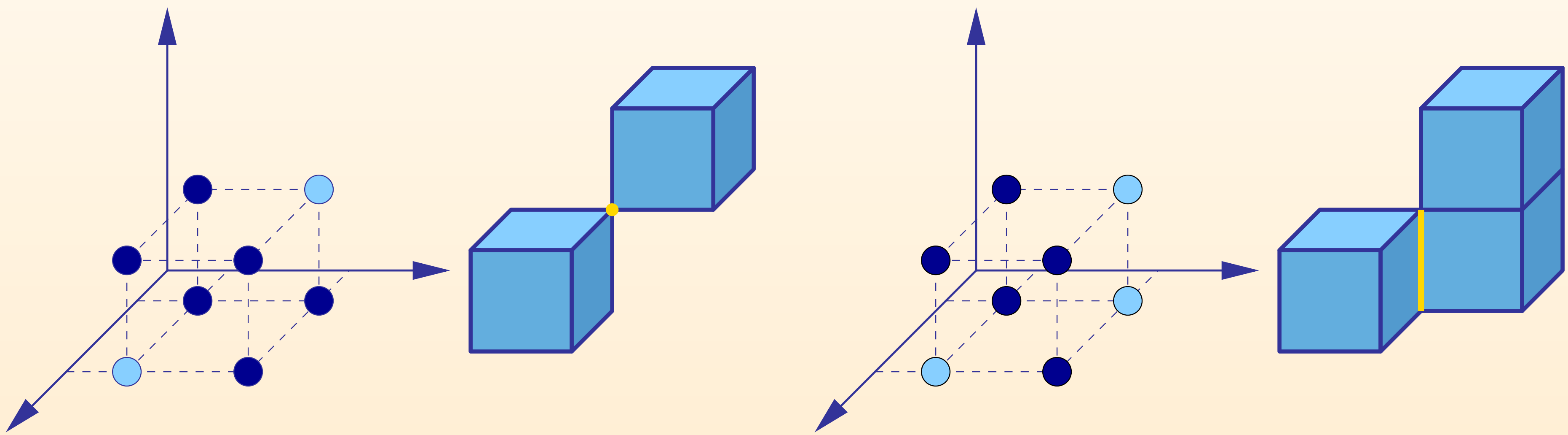

However, the conversion of one background point into a foreground point **always** gives rise to **a new** critical configuration 1.

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By observing the picture below, we see that a critical configuration 2 of type 2 can **only be removed if** one (or both) of its two background points is made into a foreground point.

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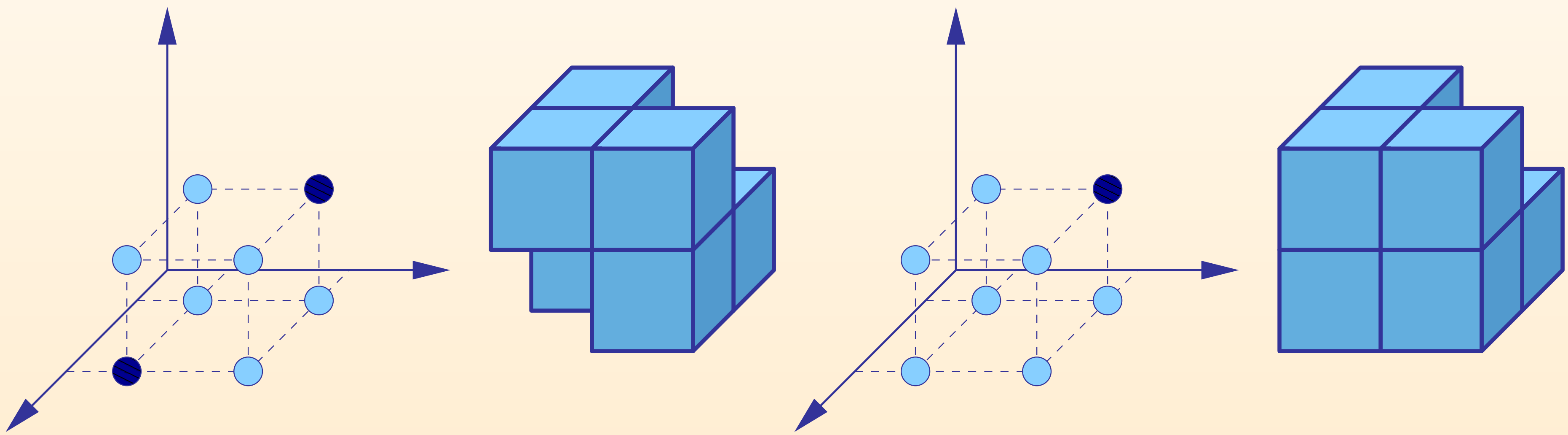

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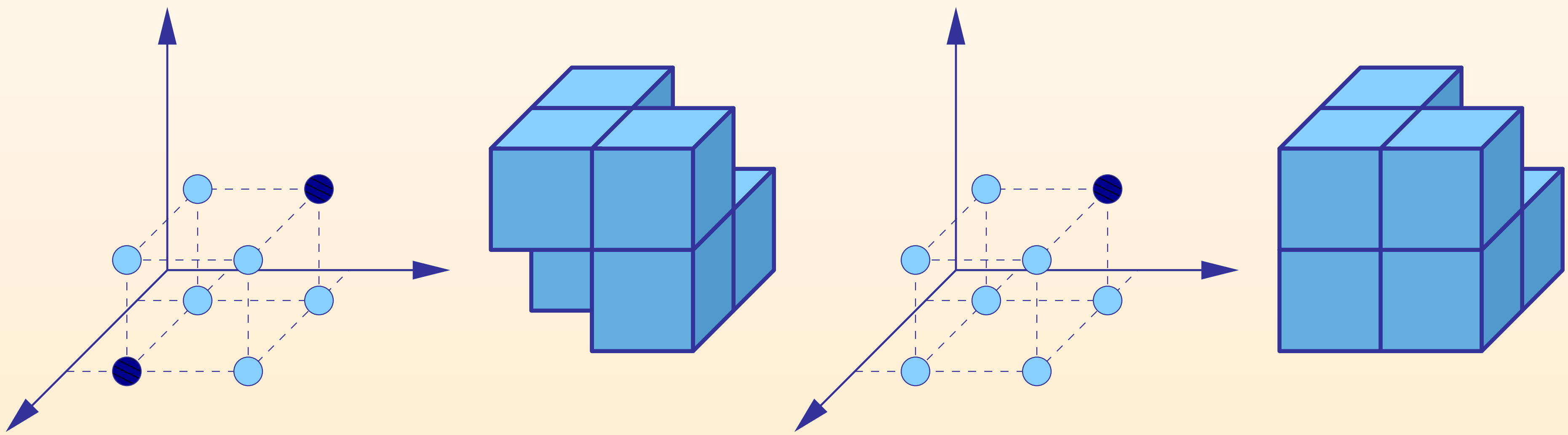

This case is similar to the case of removal of an instance of (C1).

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If CC is an instance of $(C1)$ or an instance of $(C2)$ of type 2, then S has exactly two background points of $D \setminus P^i$. Otherwise, CC is an instance of $(C2)$ of type 1 and therefore S contains exactly six background points of $D \setminus P^i$.

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We know that the conversion of any point of S into a foreground point eliminates CC. However, such a conversion may or may not create a new configuration.

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Let $S' \subseteq S$ be the subset of points q of S that can be converted into foreground points without giving rise to a new critical configuration in $(D, X \cup (P^i \cup q))$.

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By applying any of these rules, our algorithm is guaranteed to eliminate CC. Furthermore, a new critical configuration is created if and only if the third rule is applied. This new critical configuration will be eliminated in the next iteration.

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- Since background points are never removed from P, a critical configuration that is eliminated at iteration i cannot show up again at iteration $j > i$.

The correctness of our algorithm follows from the fact that it terminates, as $(D, X \cup P^i) = (D, X \cup P)$ contains no critical configurations after the last iteration i of the algorithm.

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Note that the size of P is bounded above by the number of **all** critical configurations eliminated by the algorithm, as each point inserted into P eliminates at least one critical configuration.

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Theorem. *Let (D, X) be a 3D binary digital image. If there are exactly m instances of critical configurations in (D, X), then the* **expected value** *$E[t]$ of the number t of new critical configurations created by the repairing algorithm described before on input (D, X) is less than $m/2$.*

The Repairing Algorithm

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Theorem. *Let (D, X) be a 3D binary digital image. If there are exactly m instances of critical configurations in (D, X), then the* **expected value** *$E[t]$ of the number t of new critical configurations created by the repairing algorithm described before on input (D, X) is less than $m/2$.*

Since the size of P is bounded above by the number of critical configurations in (D, X), the above theorem tells us that the expected value $E[|P|]$ of the size of P is $E[|P|] < 3m/2$.

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So, in principle, if the number of critical configurations is small compared to the number of points of the image, our algorithm is expected to produce a well-composed image that is similar to the input ill-composed one.

Experimental Results

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Two other images are real lung images corresponding to the inspiration and expiration stages of lung motion.

The sixth image is a male thorax image from the dataset of the Visible Human Project (*www.nlm.nih.gov/research/visible*).

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For each of the six images, the average size of P was at most 0.32% of the size of the image.

The average number of new critical configurations is no larger than $0.2 \cdot m$, where m is the number of critical configuration in the input image.

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For each of the six images, the average size of P is smaller than m, which means that the size of P and the size of the smallest possible P are close.

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Code available at `http://www.seas.upenn.edu/~marcelos/wellcomp.html`.

Image	# C. Conf. 1)	# C. Conf. 2)	Avg. $|\mathbf{P}|$	Avg. # New C. C.
Brain (White Matter)	2538	418	1833.5	249.9
Brain (Grey Matter)	9688	1316	6834.2	2115.9
Brain (CSF)	8574	676	5303.3	787.1
Lung (Inspiration)	1908	128	1213.7	288.4
Lung (Expiration)	3284	237	2068.7	483.2
Thorax	1962	84	1123.7	143.8

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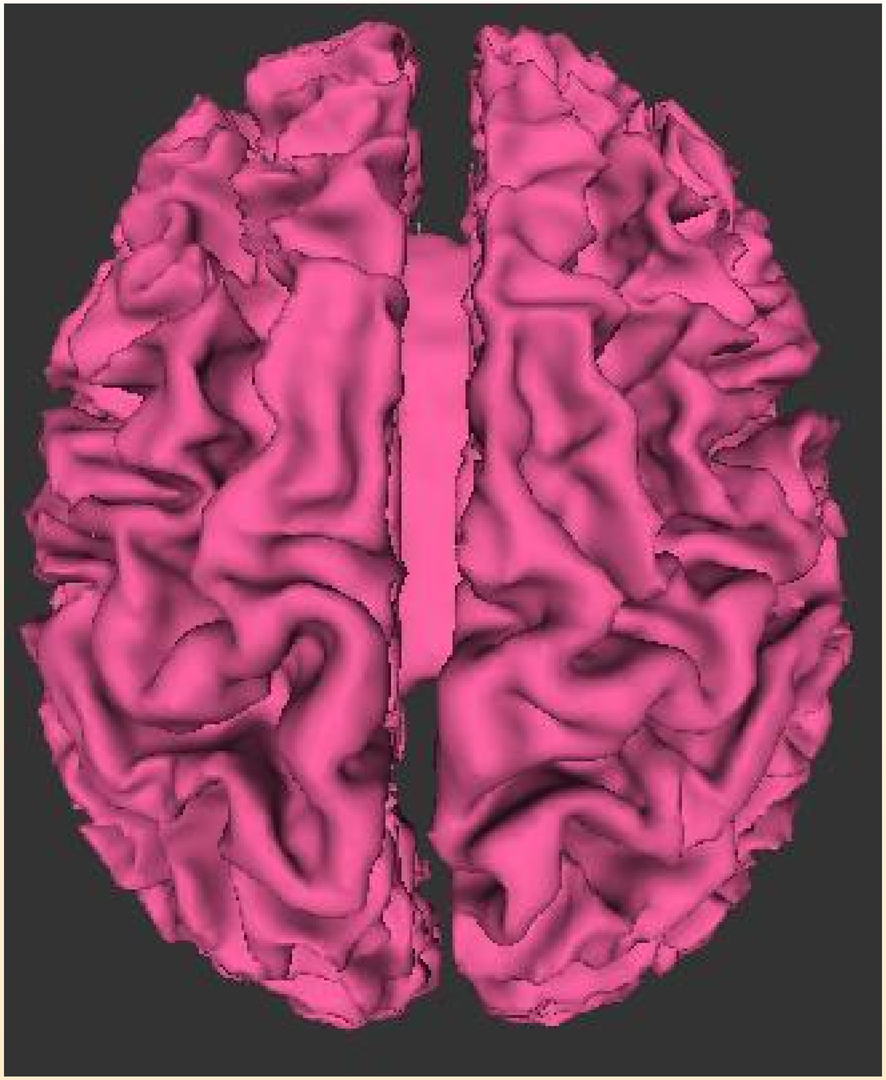
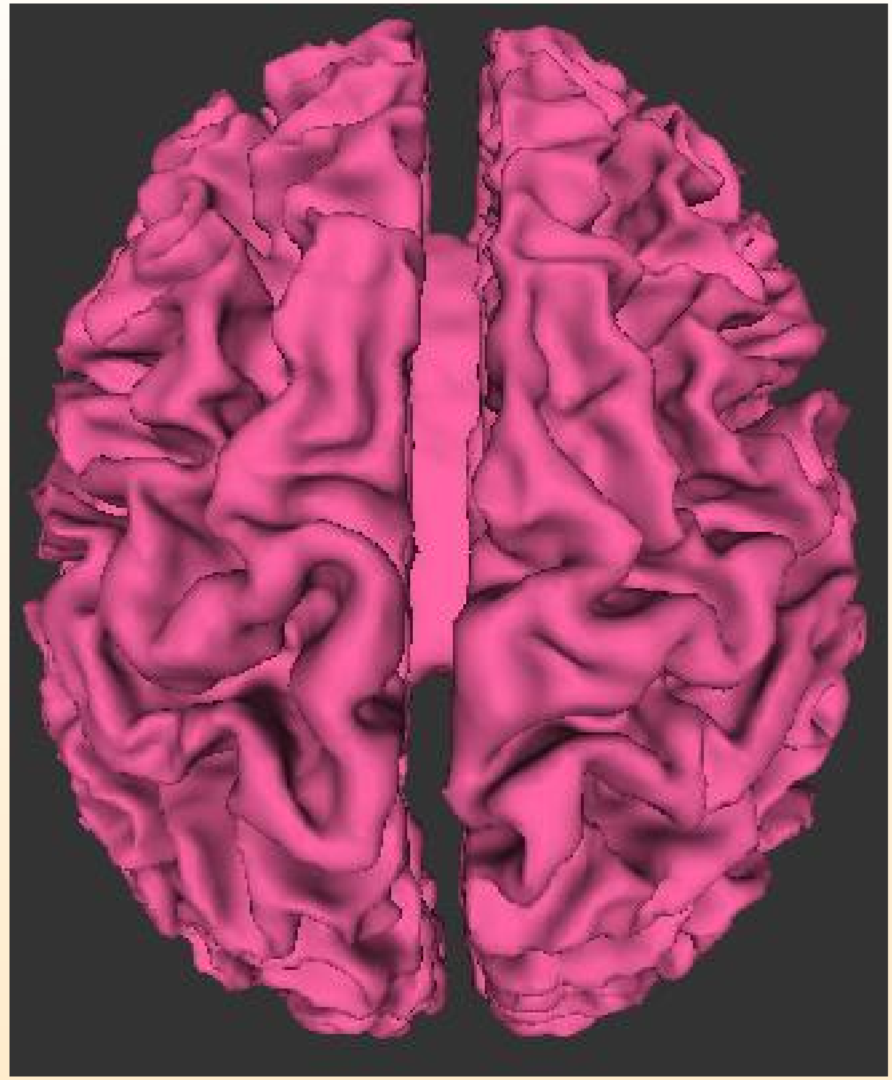
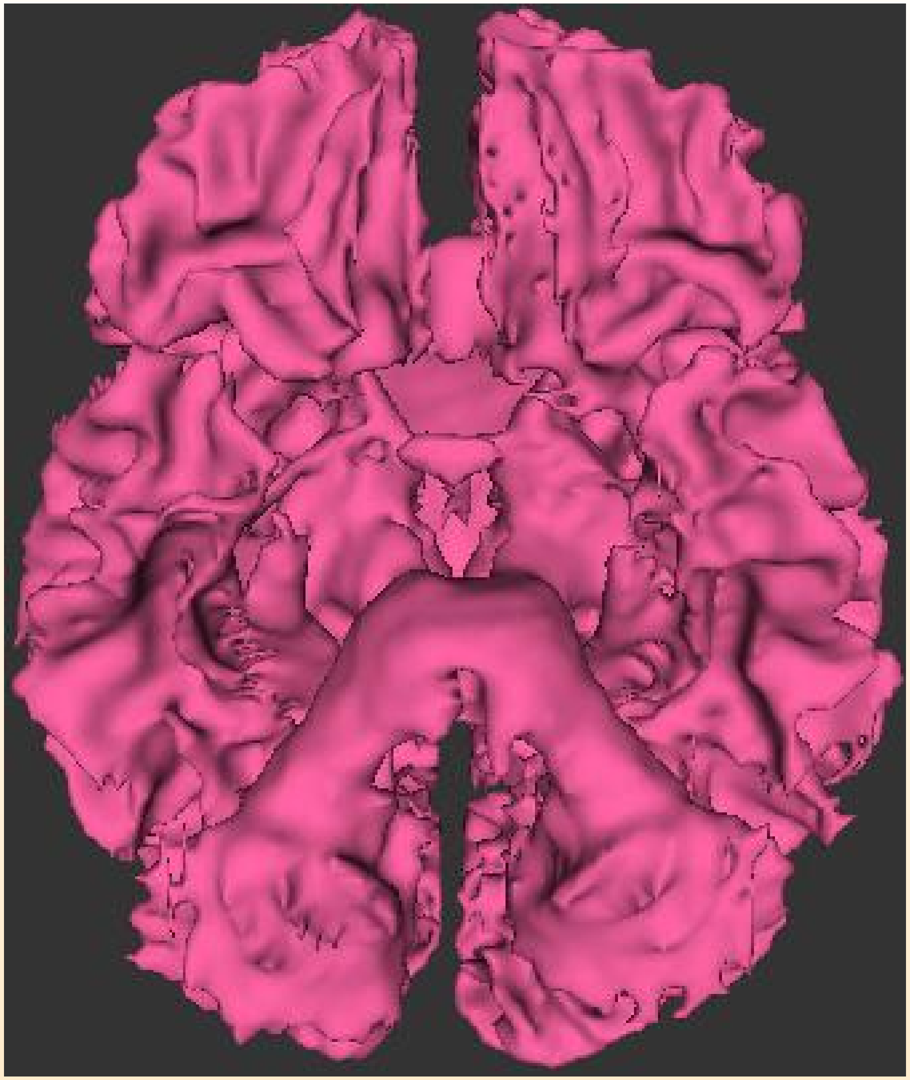


From ill-composed image

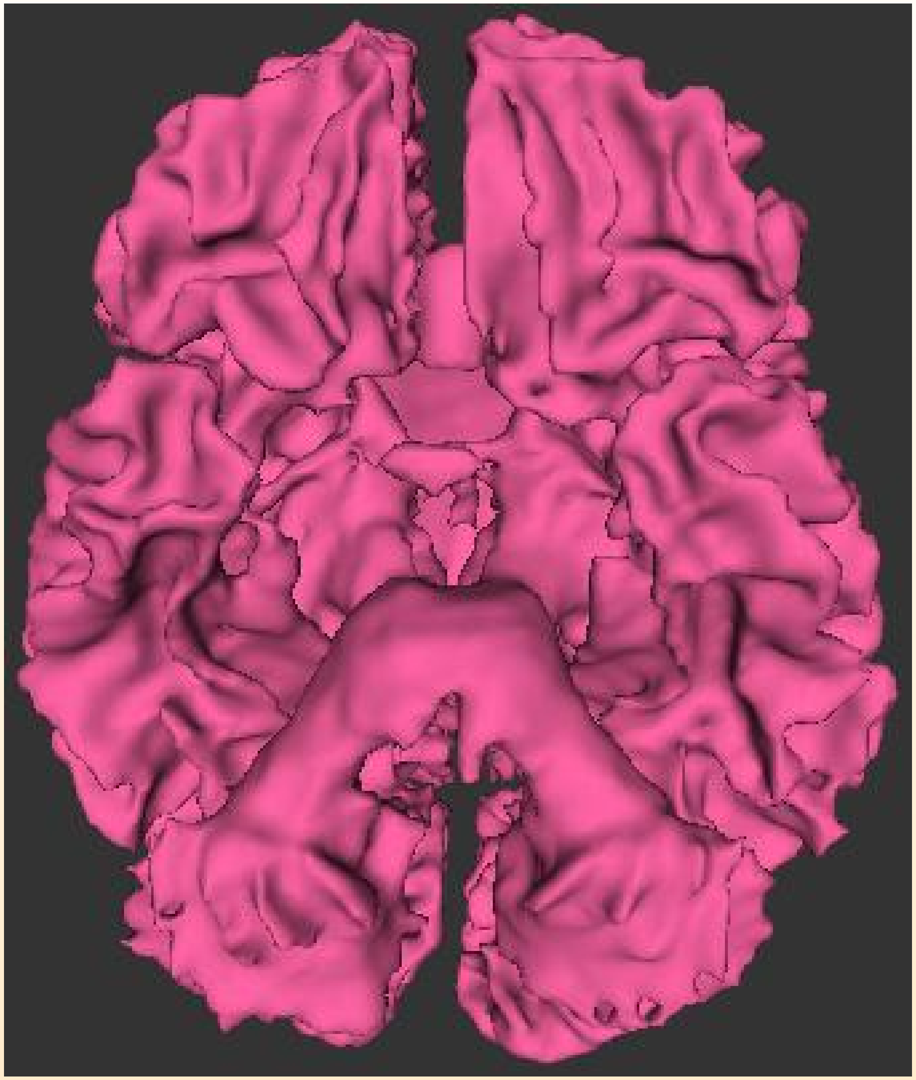


From well-composed image

Experimental Results



From ill-composed image



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Conclusions

We provided a simple randomized algorithm for repairing ill-composed images in linear time in the number of points of the image.

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For future work, we intend to (1) investigate the existence of a linear time algorithm for finding the smallest set P; (2) include a "look ahead" mechanism in our algorithm to reduce the size of P; and (3) extend our repairing algorithm to handle multicolor images.