Perfect Matchings on 3-Regular, Bridgeless Graphs

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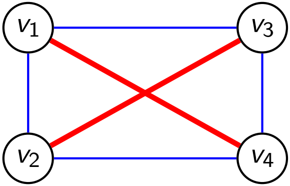

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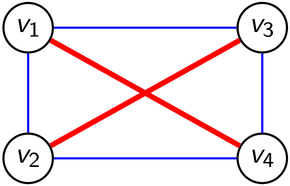

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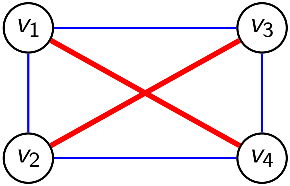

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- Vertices of matching edges are said to be *matched* or *covered* by M.

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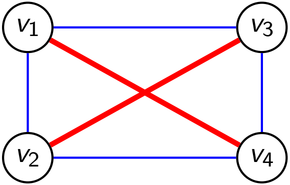

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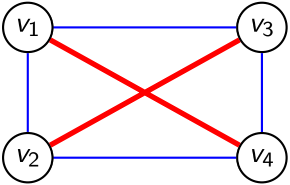

- A matching M on G is said to be *perfect* if and only if all vertices of G are matched by M. So, the matching M in the above example is perfect.

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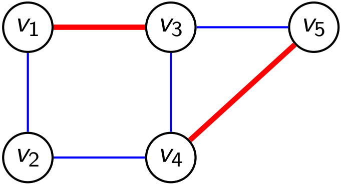

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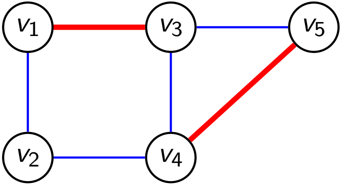

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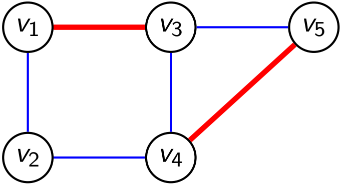

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- There are graphs that always admit perfect matchings (and they show up in graphics applications): the so-called *3-regular and bridgeless graphs*.

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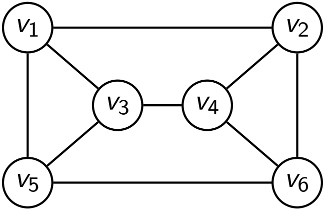

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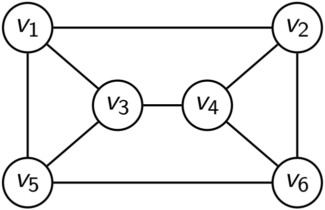

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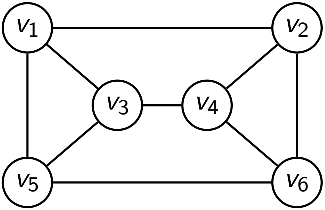

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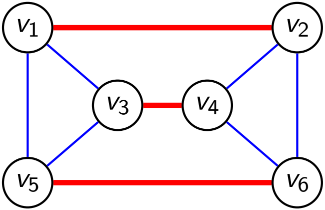

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- G is bridgeless if and only if G has no bridges.

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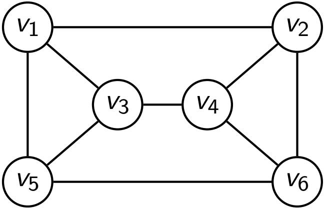
  A 3-regular, bridgeless graph $G = (V, E)$.



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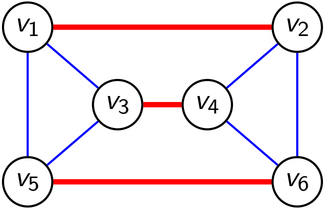

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- **Assumption**:
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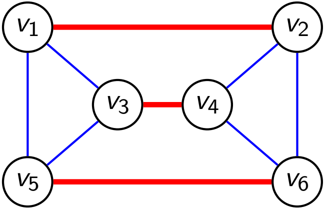

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- Note: the *best known upper bound* has been out there for about 35 years!

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Teaser

What can you say about G if no edge of G is simple?

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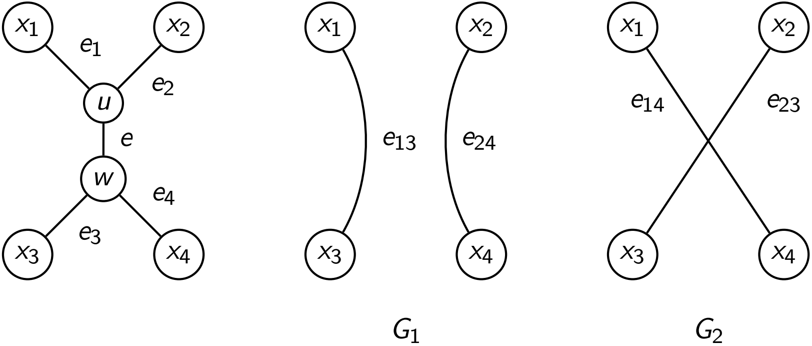

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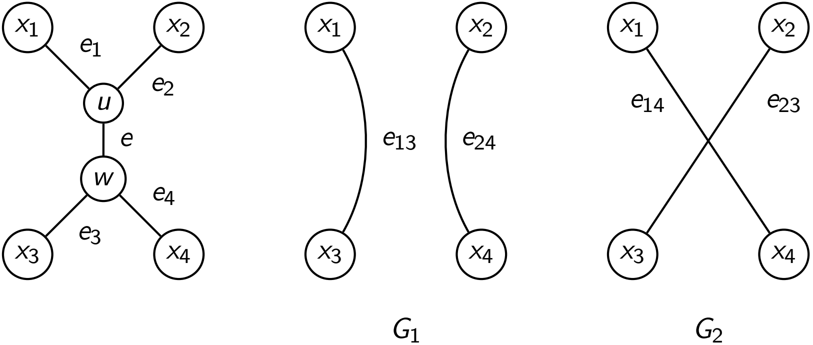
At least one of G_1 or G_2 is connected, 3-regular, and bridgeless.



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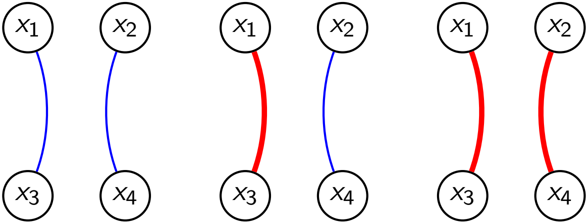

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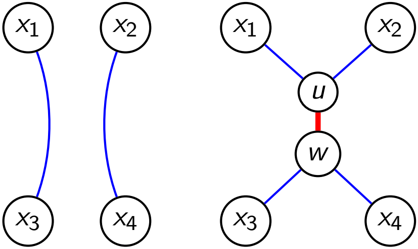

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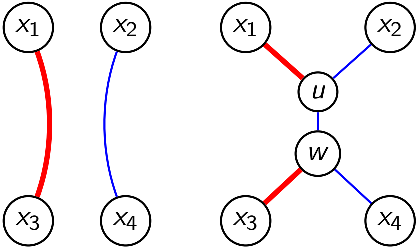

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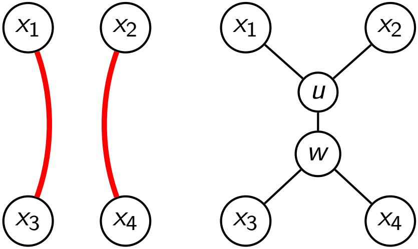

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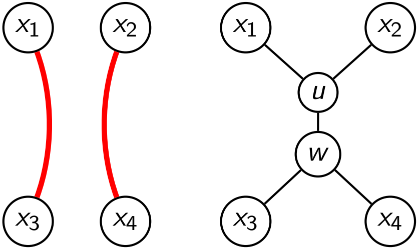

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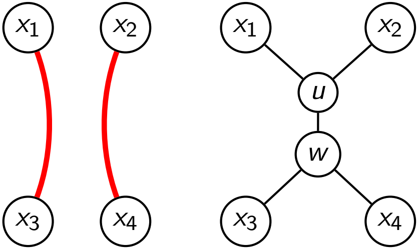

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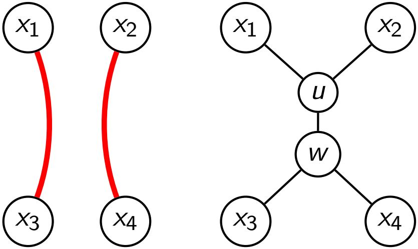

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- So, reverse an alternating cycle with either $\{x_1, x_3\}$ or $\{x_2, x_4\}$.

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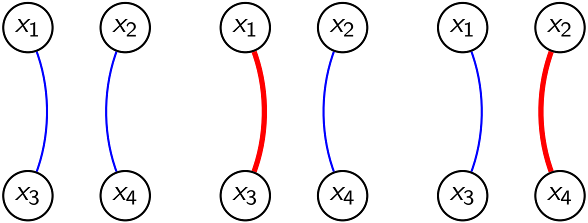

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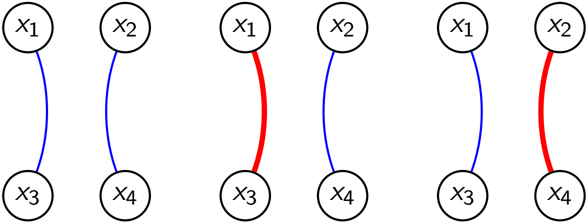

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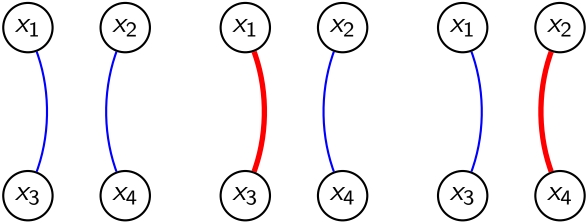

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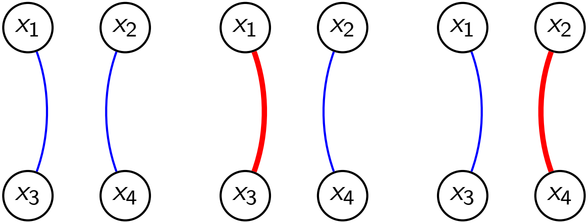

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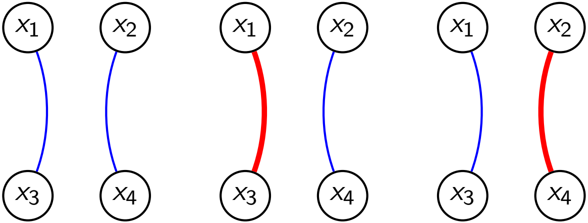

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  - What is the cost?

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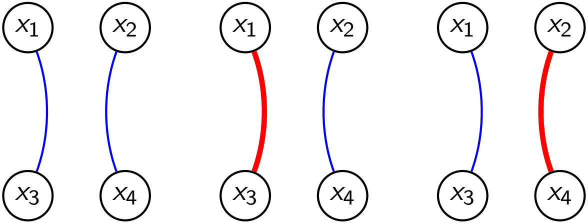
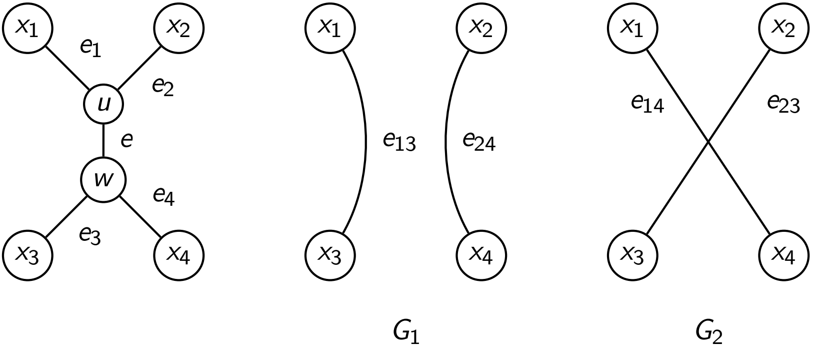


- Good, but...
  - How can we find the alternating cycle at first place?
  - Finding an augmenting path on $G_1 - \{x_1, x_3\}$ or $G_1 - \{x_2, x_4\}$.
  - What is the cost?
  - $\mathcal{O}(m)$ amortized time (**be careful here**!)

Frink's Algorithm

- A "little" problem remains unsolved:



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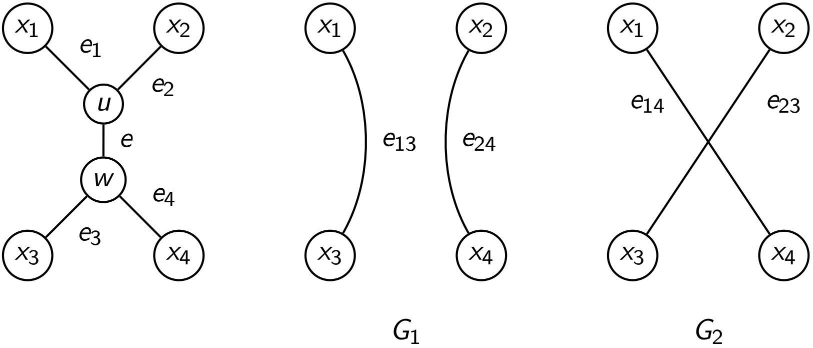
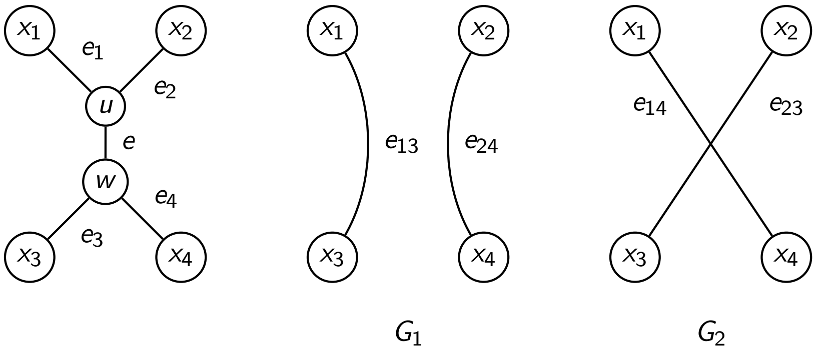
- A "little" problem remains unsolved:
- How can we decide which graph (G_1 or G_2) satisfies Frink's theorem?



Frink's Algorithm

- Counting biconnected components of G_1 and G_2.



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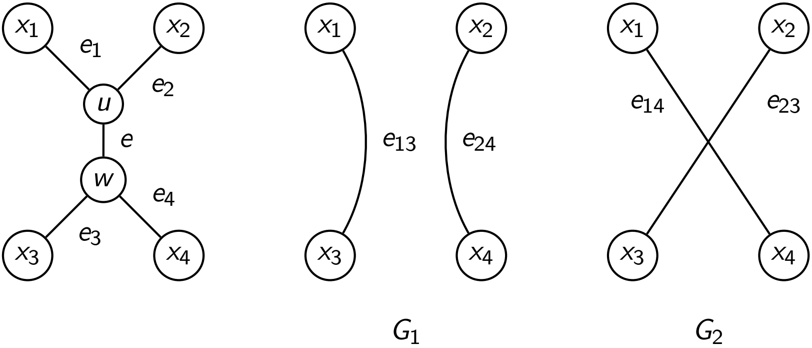

Frink's Algorithm

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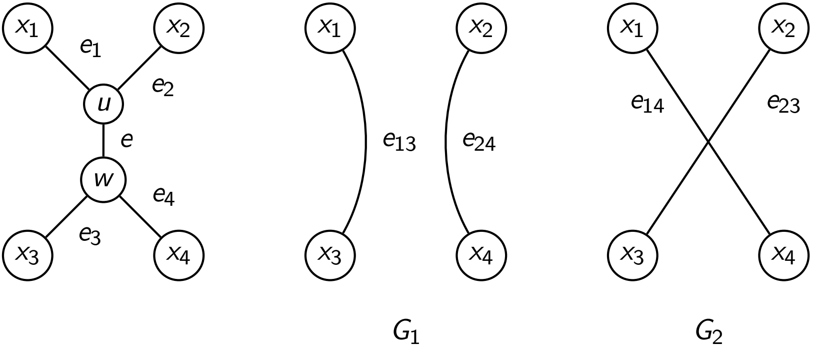
- So, we can compute a perfect matching on G in $\mathcal{O}(n^2)$ time.



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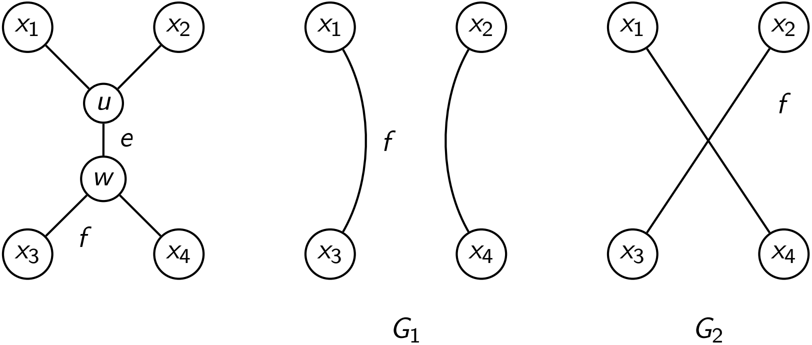

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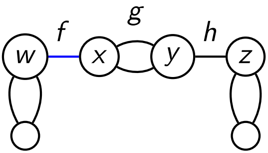

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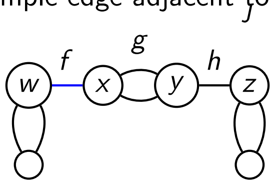

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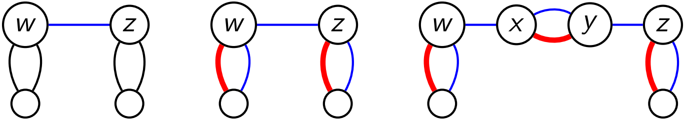

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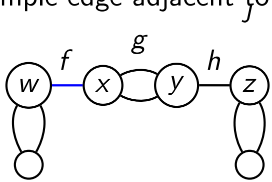

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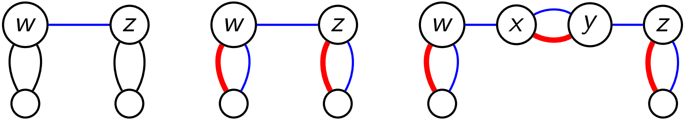

- So, each reduction takes constant time now!

A Faster Biconnectivity Test

- Recalling...
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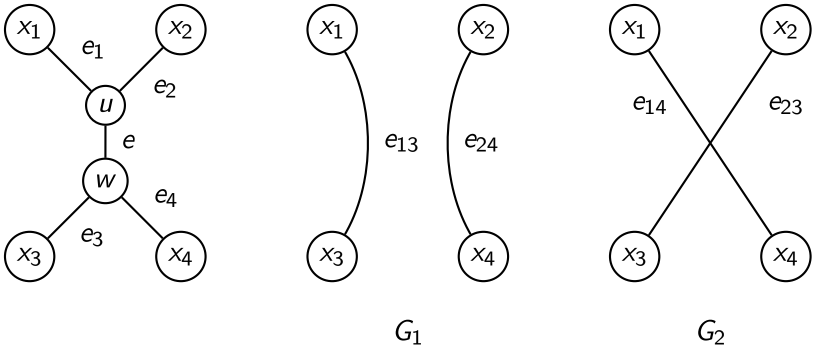

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- Can talk about Diks and Stanczyk's algorithm some other time...